BVIN: The Bitcoin Volatility Index

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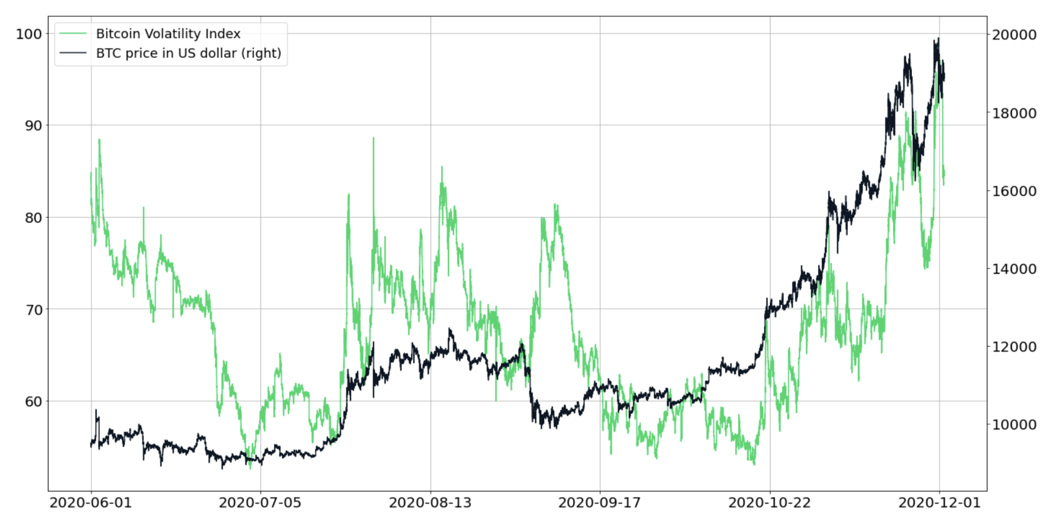
An implied volatility index is a single number that summarises the entire implied volatility smile at a fixed maturity. Its value is calculated from the current prices of European options of different strikes, and the index formula is derived in such a way that it represents the fair value of a variance swap.

This document explains how to calculate the CryptoCompare bitcoin volatility index, BVIN, which is based on the same fair-value formula as the other VIX indices. The highest recorded value of BVIN was 170% which occurred between 11 and 13 March 2020, as the Covid-19 pandemic news sent shock waves into US stock prices and bitcoin lost any appearance of being a safe-haven asset once and for all.

Figure 1: BVIN and Bitcoin Price (minute level) 1 June – 12 December 2020

BVIN (left-hand scale, green) and the USD price of bitcoin (right-hand scale, black)



Minute BVIN (left axis) and Bitcoin price in USD (right axis) from the 1st of June 2020 to the 2nd of December 2020

Figure 1 shows how the BVIN has changed since June this year, as the price of bitcoin approached its highest level since December 2017. We quote the index every 15 seconds, but the BVIN and bitcoin price series here are depicted at the minute frequency between 1 June and 12 December 2020. At the end of November the bitcoin price (in back, right-hand scale) climbed to nearly $20,000 as BVIN (in green, left-hand scale) rose from its long-term average level of around 60% to exceed 90%. When it spikes, BVIN displays its sensitivity to investor's demand for out-of-the-money (OTM) put options on fears of an imminent crash in the bitcoin price. A feature that is not shared with equity markets is that the implied

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volatility of bitcoin is not always negatively correlated with returns. The VIX index peaks when the S&P500 index crashes, but remains relatively stable during a bull market for US equities. By contrast, when bitcoin prices are already near an all-time high, further rises can induce fear in option traders who then put pressure on OTM put prices – and at such times the BVIN can actually increase on further price rises. Otherwise, during the more 'normal' bitcoin markets, a negative correlation between BVIN changes and bitcoin returns is more typical.

1 Introduction

The model described in this methodology document is not new. Indeed, variance swaps on equity indices were first introduced by Goldman Sachs in the 1990's and their fair value was derived by Demeterfi et al. (1999). The corresponding implied volatility index (VIX) for a fair-value S&P 500 variance swap rate was termed 'The Investor Fear Gauge' by Whaley (2000). Since then many other VIX-type implied volatility indices on equities, bonds, commodities and currencies have been quoted by the CBOE and other derivatives exchanges, and used as settlement prices for exchange-traded futures and options contracts. Also, in order to study the determinants of the bitcoin variance risk premia, not just at 30 days but for an entire term structure, constant-maturity BVIN indices for a variety of maturities between 7 and 90 days were originally computed at the 15-minute frequency by Alexander and Imeraj (2021).

In this document we state the general formulae for an n-day BVIN, for any positive n but we focus on a 30-day BVIN and emphasise the necessary data filtering of Deribit bitcoin option prices with maturities either side of 30 days. Initial calculations generate a 30-day index every second, then this is filtered for outliers before streaming it live at the highest possible frequency – currently it is quoted every 15 seconds.

As mentioned above, we compute a fair-value variance swap rate formula already used by the CBOE for many years to quote the S&P 500 volatility index. As such, the bitcoin VIX itself is not an indicative value for a tradeable instrument. However, just like the original VIX, the BVIN could be used as the settlement price for bitcoin volatility futures contracts and, if these futures become actively traded, their market prices may be used as a means to construct indicative values for a diverse set of leveraged, direct and inverse bitcoin volatility ETFs and other exchange-traded products.

The BVIN value at time t is denoted V_t^{30}. It changes every time any required data input changes. The key interim calculation for the index focuses on computing a (synthetic) 30-day implied variance. Once derived, this implied variance is easily transformed into an index representing a volatility, measured as an annualised percent. We just use the well-known square-root-of-time rule, i.e. we multiply the square root of this 30-day implied variance by $\sqrt{365/30}$.[2] A value of 80% for the BVIN, for example, means that on average, bitcoin option traders anticipate that bitcoin spot will have a volatility of 80% over the next 30 days. A volatility of 80% over the next 30 days means that the distribution of 30-day log returns has a standard deviation of $0.8 \times \sqrt{30/365} = 23\%$.[3] What does this standard deviation tell us? Making some

[2]Equivalently, we take the square root of the product of 365/30 times the 30-day implied variance. Note that bitcoin is traded 24/7 so we use 365 calendar days rather than 252 trading days in this calculation. More generally, to construct an n-day BVIN we use $\sqrt{365/n}$ times the square root of the synthetic n-day implied variance.

[3]This calculation is based on the square-root-of-time rule which, in turn, assumes that log returns are independent and identically distributed. The factor 365 converts an annual figure to a daily figure – remembering that crypto

approximations,[4] we can say that bitcoin option traders are 95% sure that the 30-day return on bitcoin will be somewhere between −0.37% and +0.58%. For instance, if the spot price is $10,000 now, a VIX of 80% implies a 95% confidence interval of $[\$6,300, \$15,800]$ for the price 30 days from now, based on the views of option traders, who are usually the most informed.

The synthetic 30-day implied variance is calculated from two other implied variances, at maturities T_{1t} and T_{2t}. These implied variances are derived from real traded prices of standard European options. The shorter maturity T_{1t} is the actual traded maturity closest to but *less* than or equal to 30 days. Similarly, the longer maturity T_{2t} is the actual traded maturity closest to but *greater* than 30 days.[5]

Example 1: At 08:00 UTC on 15 June 2020 the two adjacent maturities are for option expiry dates of 26 June (expiring at 08:00 exactly 11 days from 16 June) and 31 July 2020 (expiring at 08:00 UTC exactly 46 days from 16 June). Thus on 15 June the short maturity was $T_{1t} = 11/365$ and the long maturity was $T_{2t} = 46/365$. But on 16 June, $T_{1t} = 10/365$ and $T_{2t} = 45/365$.

2 Data Filtering

Using the Derbit exchange websocket, CryptoCompare live streams this index by filtering the prices of Derbit put and call options. Deribit bitcoin options are standard European puts and calls, so they have two characteristics, their strike K and maturity T. The settlement price for Deribit bitcoin options is the reference spot rate denoted by BTC.[6] This is an equally weighted average of the bitcoin (XBT) USD rates traded on several exchanges.[7] Maturities range from a few days to several months and strikes are denominated in USD. For constructing a 30-day index we use options of two maturities T_{1t} and T_{2t} straddling 30 days and at this part of the term structure listed strikes are usually at increments of $250, $500 or $1000.[8]

We use the BTC value as the *separation strike* K_0 to separate the out-of-the-money (OTM) from the in-the-money (ITM) options. We only use OTM options in the VIX calculation described in Section 3, i.e. we use mid-prices of puts below the separation strike and mid-prices of calls above this strike. OTM options have much lower prices than ITM options of the same strike and as a result they are usually traded more often. By using only OTM options for each strike prices are less likely to be stale.

The liquidity of the Deribit option market determines the strike range for the option prices taken into

assets are traded 24/7 – and the factor 30 makes the daily figure into a 30-day one. Just like interest rates, we always quote volatilities as an annualised percent. This is standard in all financial markets because it allows for easy comparison between volatilities at different maturities.

[4]Under the typical log-normal price assumptions, option traders believe with 95% certainty, that, 30 days from now, the bitcoin log price will deviate about 2 standard deviations from the current log price. Taking exponentials gives the result.

[5]More generally, for a VIX index V_t^n of constant maturity $T = n/365$ we have $T_{1t} \leq T < T_{2t}$ with T_{1t} and T_{2t} being the closest maturities actually traded at time t. As per convention for option pricing models, we measure maturity in years, not days.

[6]See Deribit BTC option specification.

[7]See Deribit general information for precise details of its calculation.

[8]More generally, strikes for Deribit options are listed in increments of between $100 and $2000, with the increments and traded strike ranges both increasing with the option's maturity.

the calculation, which is controlled by a parameter δ. We denote the mid-price of a put option with strike K and maturity T at time t by $P_t(K,T)$ and the mid-price of a call with the same characteristics by $C_t(K,T)$. In addition to the value of BTC at time t, denoted S_t in the following, the data required for the VIX calculation at time t are prices of OTM options at T_{1t} and T_{2t}. More precisely, for $j = 1, 2$ and for some fixed value of δ with $0 < \delta < 1$ we input:

1. Prices of put options $P_t(K, T_{jt})$ for all strikes K such that $(1-\delta)S_t \leq K \leq S_t$; and
2. Prices of call options $C_t(K, T_{jt})$ for all strikes K such that $S_t \leq K \leq (1+\delta)S_t$.

The value of δ determines the strike range $\{(1-\delta)S_t, (1+\delta)S_t\}$. In highly liquid option markets the accuracy of a VIX as a fair-value swap rate increases with δ but if δ is too large then stale option prices could distort the calculation. A high value for δ allows deep OTM options into the calculation and these are not traded as frequently as at-the-money (ATM) options. So, when the VIX is updated the prices of some deep OTM options could be stale, especially if trading is generally thin. On the other hand, selecting a value for δ that is too low yields a significant truncation error, i.e. a VIX that underestimates the fair value of a variance swap. Balancing the need for liquid prices against the size of truncation error, and given the expanding size of the Deribit bitcoin options market, we currently apply a default setting of $\delta = 0.75$.

Example 2: Suppose BTC is $S_t = \$10,000$ and that we set $\delta = 0.75$. In this case the strike range will be $[\$2,500, \$17,500]$. When the minimum strike increment is \$250 we use a maximum of $k = 61$ option prices in each implied variance calculation, with $K_1 = \$2,500$ and $K_{61} = \$17,500$ with K_{31} being the ATM strike. For strikes K_i with $i = 1, \ldots, 30$ these will be OTM puts and for strikes K_i with $i = 32, \ldots 61$ these will be OTM calls.

Having fixed the strikes for the OTM options that may be used in the calculation, we then apply two volume filters. First, we check the trading volume for each strike during the last 24 hours. If no lowest-strike puts were traded we delete this strike from the calculation, and similarly for the highest-strike calls. For zero trading volume on an interior strike we interpolate a synthetic price from adjacent liquid prices.[9]

Secondly, we use a similar linear-interpolation-over-variance to 'blend in' the new issue from Deribit that has maturity greater than the current short maturity and less than 30 days.[10] These options will typically (but not always) take the place of the short maturity instruments. For instance, if we are currently using 11 days for the short maturity as in Example 1 and Deribit issues new 21-day options, then these take the place of the short straddling maturity. To avoid noticeable jumps, which occur whenever the shorter end of the bitcoin implied volatility term structure is in steep contango or backwardation, we

[9] An interior strike is a put strike that is not the lowest in the range, or a call strike that is not the highest. The interpolation is linear over implied variance. In other words, we convert each price to an implied volatility using the Black-Scholes formula, square it and then linearly interpolate between these two variances. The reverse transformation converts the interpolated variance back into a price for use in the VIX formula. We use this method of interpolation to 'blend in' prices of new issues too. We do not interpolate option prices over two deleted adjacent strikes. Instead we truncate the strike range at the previous strike, i.e. the traded strike that is closer to ATM.

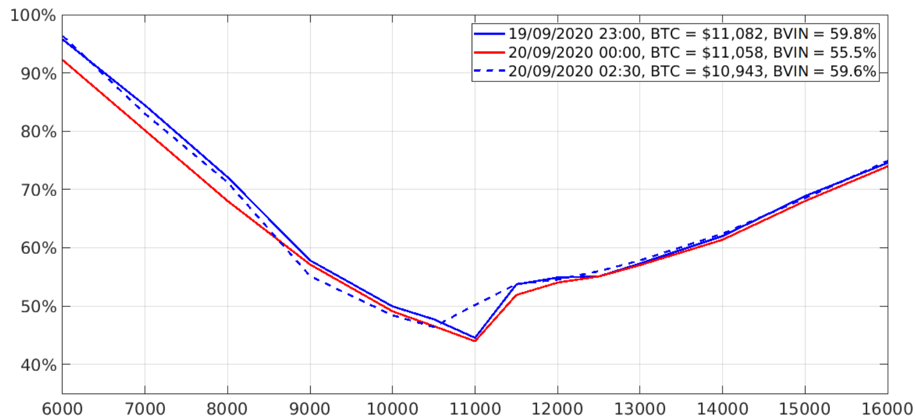
[10] See BTC options issuance schedule for details of Deribit's policy in November 2020.

interpolate between the old and new short maturities over a period of 7 days, taking 1/7 more on the new issue each day. Very occasionally, Deribit are late to issue a new monthly or quarterly option, in which case we interpolate to 30 days using the first available option with maturity greater than 30 days (and the shorter maturity, of course). Then, when the scheduled option is issued we blend this in over 7 days, i.e. we interpolate between currently-used long maturity and the new issue which is less than that maturity but greater than 30 days.

These volume-based data filters are still not sufficient to remove all the idiosyncratic behaviour that can occur when option prices are taken into a live data stream. Isolated price dislocations on one or two strikes can persist for more than a minute, although these are quite uncommon and we witness them less as the market matures. Recently, we have only witnesses them on deep OTM options during the hours around midnight UTC. It is notable that midnight UTC is taken as the daily close price in most of the daily-quoted bitcoin price indices. It is entirely because of such outliers that we only report the index every 15 seconds. We need to perform an outlier check and removal after every calculation. The outlier detection code is fairly standard so specific details are not included here.

Figure 2: Temporary Deepening of the Skew at 23:00 UTC on 19 September 2020

Implied volatility by strikes: at 23:00 UTC on 19 September 2020 (blue) and 1 hour later (red).



The final feature we discuss here is the effect on the BVIN of the demand and supply pressures on deep OTM put options. For those holding long positions on bitcoin, such options are an inexpensive form of insurance against large price falls. Now, the prices of such options can be very easily affected by a few ‘well placed’ limit orders. And it is these options in particular that have the greatest effect on the BVIN – as we shall see in Section 3.

To exhibit this happening in practice, Figure 2 depicts three 30-day implied volatility smiles, at 23:00 UTC on 19 September 2020, one hour later, and then at 02:30 on 20 September 2020. These smiles are derived by inverting the well-known Black-Scholes options pricing formula to back out the volatility that is implied by current mid-prices of Deribit bitcoin options. For each strike, at each time, the implied volatilities for the two maturities straddling 30 days are derived and then a linear-interpolation-over-variance is applied to derive synthetic smiles at exactly 30-days maturity. The three smiles in Figure 2 display a typical skew feature where investor’s fears of price falls increase the demand for deep OTM puts.

This extra demand pushes up the prices (and hence also the implied volatilities) of deep OTM puts so low strike volatilities are higher than high-strike volatilities of similar delta.

Unlike the occasional and temporary demand pressure on deep OTM puts, the kink in the smile at the 11,500 strike here doesn't lead to any jumps in the BVIN. Indeed, it is a common skew feature which can occur either side of the separation strike. In this case there is extra demand on the just-OTM calls which indicates a bull market for bitcoin, and in a bear market we could witness a similar upward kink below the separation strike on the just-OTM puts.

We shall see below in Section 3 that OTM put prices have much more influence on the BVIN than OTM call prices, and the lower the strike the greater the effect. Thus, when a situation occurs such as that illustrated in Figure 2, there can be a noticeable jump of a few percentage points in the BVIN – for instance, at 00:00 the BVIN dropped from almost 60% to 55%. But it was only temporary, the BVIN returned to its previous level by 02:30 on 20 September. This downward jump and reversal was caused by a temporary release of the usual demand pressure on OTM put prices. Specifically, an order book analysis of the put options at strikes 8000 and below is likely to show some unusually large and temporary bid volumes just below the ticker price, which brings the prices of those options down, but only temporarily.[11]

3 Index Calculation

In this section, rather than use different notation for puts and call, we use $\mathcal{Q}_t(K_i, T_{jt})$ to denote the mid price at time t of the OTM option with strike K_i, for $i = 1, \ldots, k$ and maturity T_{jt}, for $j = 1, 2$. Then, at time t, the implied variance of maturity T_{jt} is an approximation for the fair value of a variance swap, viz.[12]

$$\theta_{jt} = 2 \sum_{i=1}^{k} K_i^{-2} \mathcal{Q}_t(K_i, T_{jt}) \Delta K_i, \tag{1}$$

where $\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$ and k is the number of strikes for the options used in the calculation, as explained in Section 2.[13]

Next, we use linear interpolation between θ_{1t} and θ_{2t} to obtain a 30-day implied variance (or, more generally, an n-day implied variance) and quote the result as an annualised percent. The interpolation coefficient is

$$\omega_t^{30} = \frac{m_{2t} - m_{30}}{m_{2t} - m_{1t}}, \tag{2}$$

where m_{jt} is the number of minutes until maturity T_{jt} for $j = 1, 2$ and m_{30} is the number of minutes in

[11]Whether this order-book behaviour has any effect on other bitcoin spot and derivatives markets, is an interesting question for further research. We selected the case depicted in Figure 2 because we often need to filter BVIN for distortion effects between the hours of 23:00 - 00:00 UTC and/or 00:00 to 01:00 UTC. For instance, even though the MOVE contracts on FTX are not based on option prices, it is possible to hedge them using straddles that are traded between 23:00 UTC and 01:00 UTC.

[12]We omit the division by maturity T_{jt} here, although the implied variance formula is often presented in that (annualised) form. This is because the subsequent formulae are simplified when we annualise only at the end result, that is, in (4).

[13]For the first and last strike, $\Delta K_1 = K_2 - K_1$ and $\Delta K_n = K_n - K_{n-1}$.

30 days.[14] Then we apply linear interpolation to obtain the 30-day implied variance:

$$\theta_t^{30} = \omega^{30}\theta_{1t} + (1-\omega^{30})\theta_{2t}. \tag{3}$$

Finally, the 30-day bitcoin VIX at time t is calculated as:[15]

$$V_t^{30} = \sqrt{\omega_t^{30}\theta_{1t} + (1-\omega_t^{30})\theta_{2t}} \times \sqrt{365/30}. \tag{4}$$

Example 3: Recall Example 1 where t is 08:00 UTC on 15 June 2020, the two straddling maturities are 26 June and 31 July 2020. On 15 June 2020 the BTC value was $S_t = \$9103.94$ and the strike range was $[\$5500, \$13000]$ based on $\delta = 0.4$.[16] The options traded at the two maturities have different liquidity on 15 June 2010, in fact one more strike is listed at the shorter maturity. The liquid strikes at each maturity are given in Table 1. If the strike is not listed it is not included in the table; and if the strike is listed but the volume at that strike is not sufficient to guarantee a reliable price, we replace the last traded price with an interpolated price. These prices are shown in red – there is only one, the 26 June 9750 call.

Applying the formula (1) to these data yields the results shown for θ_{1t} and θ_{2t}. The straddling maturities are $T_1 = 11/365$ and $T_2 = 46/365$ so the interpolation coefficient is:

$$\omega_t^{30} = \frac{66,240 - 43,200}{66,240 - 15,840} = 0.457.$$

Finally, applying the interpolation 3 and the final annualisation into a volatility (4) yields:

$$\begin{aligned} V_t^{30} &= \sqrt{0.457 \times 0.01734 + 0.543 \times 0.06556} \times \sqrt{365/30} \\ &= 72.76\%. \end{aligned}$$

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Whaley, E. (2000). The Investor Fear Gauge. *The Journal of Portfolio Management 26*(3), 12–17.

[14] More generally, for an n-day VIX, V_t^n we use m_n i.e. the number of minutes until the target constant maturity $T = n/365$, and with maturities T_{1t} and T_{2t} now selected to straddle the n-day maturity, we set

$$\omega_t^n = \frac{m_{2t} - m_n}{m_{2t} - m_{1t}}$$

where m_n is the number of minutes in n days.

[15] More generally, $\theta_t^n = \omega^n\theta_{1t} + (1-\omega^n\theta_{2t})$ and $V_t^n = \sqrt{\omega_t^n\theta_{1t} + (1-\omega_t^n)\theta_{2t}} \times \sqrt{365/n}$.

[16] In June 2020 the Deribit options market was relatively illiquid, so we used a $\delta = 0.4$ for the strike range – it is only more recently that we have raised δ to a higher default value of 0.75.

Table 1: Example BVIN Calculation on 15 June 2020

26 June 2020			**31 July 2020**		
Strike	**Type**	**USD Price**	**Strike**	**Type**	**USD Price**
6000	P	18.21	5500	P	79.66
7000	P	45.52	6000	P	109.25
7500	P	68.28	6500	P	150.22
8000	P	106.97	7000	P	202.56
8500	P	186.63	7500	P	286.77
8750	P	252.63	8000	P	400.57
9000	P	345.95	8500	P	557.62
9250	C	325.47	9000	P	769.28
9500	C	227.60	9500	C	682.80
9750	C	157.04	10000	C	507.54
10000	C	111.52	10500	C	377.81
10500	C	61.45	11000	C	289.05
11000	C	40.97	11500	C	227.60
11500	C	27.31	12000	C	184.35
12000	C	22.76	13000	C	129.73
13000	C	13.66			
	θ_{1t}	0.01733943		θ_{2t}	0.0655631
	ω_t^{30}	0.457		$1-\omega_t^{30}$	0.543